Survey on multi-robot manipulation of deformable objects

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Abstract— **Autonomous manipulation of deformable objects is a research topic of increasing interest due to the variety of current processes and applications that include this type of tasks. It is a complex problem that involves aspects such as modeling, control, perception, planning, grasping, estimation, etc. A single robot may be unable to perform the manipulation when the deformable object is too big, too heavy or difficult to grasp. Then, using multiple robots working together naturally arises as a solution to perform coordinately the manipulation task. In this paper, we contribute a survey of relevant state-of-the-art approaches concerning manipulation of deformable objects by multiple robots, which includes a specific classification with different criteria and a subsequent analysis of the leading methods, the main challenges and the future research directions.**

I. INTRODUCTION

Manipulation of deformable objects is an open problem in the field of robotics. As opposed to the well-studied framework of rigid objects manipulation, being able to predict how the object is going to behave under the effects of a certain manipulation action is a crucial and challenging aspect when dealing with deformable materials. Production of clothes and footwear, food handling, toys manufacturing and surgery are some of the applications involving these kinds of objects. One of the main interests of automatizing some of these sector's manipulation tasks is to reduce the health hazards for human workers, who have to go through uncomfortable, unpleasant and even dangerous works.

When dealing with certain types of deformable objects one can find that they are too big, too heavy, difficult to grasp, too fragile or too soft for being manipulated by a single robot. Therefore, in order to improve the performance of the robotic systems in terms of accuracy, computational cost and flexibility, multiple robotic manipulators with the same or different roles must be considered to carry out the task. Previous surveys on the topic of autonomous manipulation of deformable objects have been developed in the recent years [1] [2], and in particular the reader is referred to the comprehensive survey performed by Sanchez *et al.* [3] for a global understanding of the current state of the art in robotic manipulation of deformable objects. However, none of them is focused specifically on manipulation of deformable objects by multi-robot teams. We have studied recent approaches which are relevant in the field of manipulation of deformable objects by multiple robots, and we present a set of classifying criteria. Firstly, we classify the approaches in terms of the deformation model they utilize. We continue with a classification with respect to the dimensionality of the deformable object. After this, the methods are classified according to the control strategy they follow. A perception-based classification is performed next, and finally the approaches are classified depending on the predominant actions they deal with. Table I is presented at the end of the paper as a summary of the surveyed and classified works of the state of the art. It is worth mentioning that we consider dual-arm robots as a multi-robot manipulation system, due to the fact that each arm represents an independent manipulation unit.

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II. MODEL-BASED CLASSIFICATION

Due to the highly-dynamic behavior of deformable objects, correctly modelling deformation is one of the main concerns when handling these kinds of objects. In the past, many studies that tackled manipulation of deformable objects were based on precomputed models, but nowadays there is a clear tendency on exploiting methods which learn an online deformation model. Moreover, there are others that do not consider any deformation model of the object. This model-free perspective improves both robustness and generality of the methods. It is important to remark that the following discussion only includes methods that are implemented inside the manipulation algorithms. It does not consider models that represent the deformable object in simulation tests.

A. Methods using precomputed models

Those approaches where a deformation model is computed offline, previously to the system working in real time, are analyzed and compared in this section.

Mesh models, with either discrete (mass-spring-damper elements) or continuous (finite element method) formulation, are common candidates to represent the deformable object. Das and Sarkar [4] consider the problem of handling a 2D deformable object exhibiting rheological deformations (elasticity and viscosity properties) with a group of robotic manipulators, or a robotic hand with multiple fingers. They model the deformable object by means of a mass-spring-damper elements mesh, and deformation is controlled by applying an optimization technique over the mesh boundary nodes. Also Li *et al.* [5] utilize mesh models for the purpose of manipulating deformable objects (clothes). They have built a precomputed database which contains 3D mesh models of different kinds of garments, all of them simulated under the effects of gravity and picked up in multiple poses. When a piece of clothing is grasped for performing a manipulation task, the recognition system creates its 3D mesh model and extracts some relevant features to be compared with the ones of the precomputed models from the database. After finding a correspondence, the algorithm obtains the optimal manipulation trajectories leading to the desired state. Similarly to the previous approach, Jia *et al.* [6] follow a strategy that is based on building a visual dictionary in an offline way and using it afterwards to manipulate highly deformable materials. Their dictionary consists of a set of vectors that store visual feedback data and end-effector velocities, whose mapping is obtained by means of a training phase. Different goal configurations are computed at runtime depending on the selected manipulation task, and using sparse linear representation the velocity of the controller is computed from the visual dictionary. Manipulation tasks like folding and flattening of cloth pieces with varying material properties are performed in experimental tests, with a dual-arm robot and the dual-arm robot in collaboration with a human. Higher accuracy values may be obtained from finite element models in comparison with the mass-spring-damper mesh models. Duenser *et al.* exploit these kinds of models in [7] for performing user-specified deformations over elastic parts by means of a dual-arm robot, under the quasi-static system assumption. Their approach performs an efficient real-time optimization in which the Jacobian relating the joint angles' changes to the variations of the object's shape is continuously obtained. A different method is presented by Long *et al.* [8] for obtaining the direct and inverse dynamic models of a group of two manipulators cooperatively carrying a flexible object. This technique can be applied to rigid, articulated and flexible objects whose deformation can be expressed with a normal distribution. While the object is modeled using the generalized Newton-Euler formalism, the robotic manipulators are modeled with the rigid arm equations and the kinematic Jacobians, and the two subsystems are linked together by the wrench applied at the end effector's grasp.

The main disadvantage of the aforementioned methods is that the model has to be modified and recomputed each time the properties of the deformable object change, e.g., a T-shirt of a different size, or different material, etc.

B. Methods using learned models

Instead of using a model with fixed parameters, the following approaches focus on learning the deformation model parameters in an online manner.

With the aim of deforming 3D rheological objects, Higashimori *et al.* [9] propose a two-phase strategy in which the parameters defining the behavior of the object model are obtained previously to performing the desired deformation action. This approach is able to deform 3D objects to a desired state but only in one direction, due to the fact that it relies on a 1D four-element model (2 springs and 2 dampers) of the object. A more recent method in which a local deformation model is approximated online, through an estimation-recalibration algorithm, is proposed by Navarro-Alarcon and Liu [10] for the purpose of deforming 3D soft objects into 2D desired contours. In this approach, one active and one or more passive grippers that follow translational quasi-static movements are servo-controlled to deform the object. The object's 2D shape is represented with Fourier series, and its physical properties are completely unknown. Also in [11] Navarro-Alarcon *et al.* describe a method for deforming 3D elastic objects with two robotic arms and achieve different positions and shapes by estimating the Jacobian of the deformable object. Hu *et al.* [12] present a similar method with improved properties in terms of convergence and dynamic behavior. They estimate the object's deformation by means of an algorithm called FO-GPR (Fast Online Gaussian Process Regression), which obtains a nonlinear deformation function and updates it at each time step. In contrast to the standard GPR methods, the FO-GPR removes uninformative observation data, which allows to substantially decrease the computational cost of the algorithm and also improves the accuracy of the model. Multi-robot low-level manipulation tasks, in which the deformation function does not vary at different stages (rolled towel bending, towel folding, etc.), are successfully performed with this technique in experimental tests, even in the presence of partial occlusions. A different Jacobian-based strategy is developed by Berenson [13] in order to perform locally defined quasi-static manipulation tasks of 1D and 2D deformable objects, with a single or multiple robotic manipulators. His algorithm exploits the concept of diminishing rigidity (from the grasping point to the rest of the object) to compute an approximate Jacobian of the object, which is corrected afterwards to include excessive stretching constraints. This approach depends on manual tuning for setting an adequate evolution of rigidity, and it cannot represent properly the properties of heterogeneous deformable objects. However, a recent study by McConachie and Berenson [14] that is based on the MAB concept (Multi-Armed Bandit) solves the problem of automatically selecting an appropriate deformation model. In particular, their algorithm is called KF-MANDB, and extends the standard MAB technique to consider a nonstationary, inter-dependent, Kalman-filtered framework. They consider a set of grippers to perform a specific deformable object manipulation task, and they build a model database with approximate Jacobian models tuned with different parameter values. At the same time that the manipulation task is performed, the algorithm evaluates the utility of each model (i.e. how accurate each model is for representing the deformation state) and selects the one with the highest value.

Some additional methods that consider learning a deformation model are [15], in which the stiffness matrix of a finite element model is obtained by probing the material, and [16], in which the object's shape is approximated by Fourier series. Also a combined deformation-projection Jacobian is estimated online in [17].

C. Model-free methods

Even more flexibility can be attained, *a priori*, if the behavior of the soft object is not related to a specific deformation model. Next methods take advantage of this and develop model-free multi-robot techniques for manipulating deformable objects.

Indirect Simultaneous Positioning is a concept studied by Wada *et al.* in [18] with the goal of controlling the position of a set of points lying within the contour of a 2D deformable shape. They classify the interest points of the shape into manipulation points, to be grasped by the grippers, and positioned points, whose position is to be controlled. Then, they show two different PID control methods for achieving the control objective: a first one that relies on an approximate deformation model, and a second one for small deformations in which the deformation model is not needed. Deformations are here restricted to the plane of the 2D object. Another model-free approach proposed by Bai and Wen [19] deals with the problem of flexible load transport from a formation control perspective. They have developed a decentralized control method for the collaborative manipulation of a deformable object by a group of robotic manipulators. Despite the fact that the geometry of the object and the position of the grasping points have to be known in advance, no model of the object is considered. Some assumptions made in this study are that the deformation of the object is small and only appears in a small area around the grasping points, and also that only translational movements are performed. Within the field of robotic cutting of deformable objects, Long *et al.* [20] follow a *pressing and slicing* strategy for separating soft parts without explicitly modeling the object. Apart from the cutting robot, which is equipped with the cutting tool, a second robot is considered to provide a pulling force (whose magnitude is obtained from experiments) that reduces the necessary cutting force.

III. Object-Based Classification

A different classification of the manipulation strategies can be performed from the perspective of the manipulated object. Real deformable objects are always lying on the 3D space, but some of them are studied by omitting those dimensions which are of a much smaller magnitude than the rest. For instance, a rope can be studied by considering it a 1D entity with null cross section provided that its length is much larger than its thickness in both transversal directions. Thus, the following subsections differentiate the multi-robot manipulation approaches by the dimensionality of the deformable object.

A. Unidimensional objects

As reported before, ropes and also cables are typical examples of unidimensional objects. A recent approach by Zhu *et al.* [16] tackles the problem of manipulating 1D flexible cables to match a desired 2D contour, by means of a dual-arm robot equipped with special-purpose grippers. Inspired by [10], they represent the cable's shape with truncated Fourier series. To reach a compromise solution between accuracy, computational cost and under-actuation, only two harmonics of the series are accounted. As opposed to [10] this method considers also rotational movements, but it is not capable of predicting whether a final shape is reachable or not. The challenging task of in-air knotting of 1D ropes is tackled by Kudoh *et al.* in [21]. After extracting a set of hand motions with high reusability (*skill motions*), they develop a specialized hardware system to perform the in-air knotting task. In particular, this system consists of a dual-arm robot equipped with three-finger hands and an RGB-D camera, which is utilized to achieve the initial grasping of the rope. Also in [22], [23] a 1D rope is deformed to a target configuration by two robotic arms. The algorithm created by Tang *et al.* in [22] is called TSM-RPM (Tangent Space Mapping-Robust Point Matching), and maps the evolution of the object's curve tangents to reach the desired configuration between different initial configurations, by means of a non-rigid transformation function. The authors show in simulations how the TSM-RPM algorithm outperforms the TPS-RPM (Thin Plate Spline-Robust Point Matching) in terms of overstretching avoidance and fidelity of the final configuration with respect to the desired one.

B. Bidimensional objects

The main examples of 2D deformable objects, in terms of the number of approaches considering them, are the cloth-like ones. This type of object is unable to withstand compression forces, and usually shows high rigidity values when submitted to traction forces. Thin panels of deformable materials can be considered as 2D objects too.

Alonso-Mora *et al.* [24] propose a hybrid centralized/distributed algorithm for the transport of deformable objects by multiple robotic manipulators. While the centralized approach is considered in order to provide a global guidance to the manipulators, the distributed one enables the manipulators to move according to the global planning without explicit communication between them. This algorithm includes collision avoidance with both static and dynamic obstacles and shape preservation constraints. Three different 2D deformable objects (a foam mat, a bed sheet and a towel) are collaboratively carried to a desired position by a multi-robot team in experimental tests. Deformation is controlled here but only for the purpose of maintaining the structural integrity of the object during the transportation. Focusing also on 2D deformable objects collaborative manipulation, Langsfeld *et al.* [15] develop a multi-robot system that allows to clean plastic parts with two fixed redundant robotic arms of differentiated roles: the first arm cleans the part without deforming or breaking it while the second one holds the object in a proper position. The object is assumed elastic and is modeled with 1D finite elements whose elastic properties are updated as the cleaning task proceeds. Regrasping actions are optimized in order to minimize the part deformation and the cleaning time. A different collaborative situation in which a 2D deformable sheet is manipulated between a person (uncontrollable agent) and a dual-arm robot (controllable agent), equipped with a Kinect camera, is studied by Kruse *et al.* [25]. Initially, the opposite ends of a piece of fabric are grasped by the human and the robot, and the control goal is to minimize the amount of wrinkles and local deformations produced when the human manipulator performs local movements, that distort the initial undeformed state. Collaborative human-robot tasks, besides robot-robot ones, are considered too by Jia *et al.* in [6] for cloth folding, cloth flattening and cloth placing. The effectiveness of this method is affected by some limitations in terms of illumination and relative colors of the clothes in the 2D images.

C. Tridimensional objects

Being probably the most challenging examples of deformable objects for robotic manipulation, 3D objects range from soft foam pieces to food dough, and represent the most general case in which all spatial dimensions are accounted.

The problem of deforming 3D elastic foam parts to a desired state by multiple robots is tackled by Navarro-Alarcon *et al.* in [26]. They propose a robust vision-based controller which accounts for noise and uncertainty in the model estimation. The elastic properties of the deformable object are unknown, and therefore the Jacobian matrix is estimated online with the Broyden update rule [27]. They perform various experimental tests with one and two active manipulators. 3D soft foam parts are also considered by Long *et al.* for a cutting process in [20]. In this approach, experimental tests are carried out over the soft foam parts, that are cut to a predefined cutting depth through sequential cutting actions. The cutting trajectory is specified by a series of visual markers attached to the object. In [19] a lightweight 3D soccer ball is collaborativelly manipulated by two fixed robotic arms, once fed by a human agent. The manipulation task is divided in two steps: the first one implies statically holding and squeezing the ball, and during the second one, linear or circular trajectories are performed. All the previous approaches consider homogeneous objects whose material properties do not vary across their volume. In [17], however, Alambeigi *et al.* propose a multi-robot method for manipulating 3D compliant objects that show heterogeneous material properties. An online estimation system, based on the Secant approximation and the Broyden's method, obtains the combined deformation-projection Jacobian which allows to predict in real time the deformation and the camera parameters. Thus, their method is able to work with uncalibrated vision sensors. With regard to the control algorithm, a constrained optimization problem is solved with the previously computed Jacobian to accomplish the predefined tasks in an environment with potential disturbances.

IV. Control-Based Classification

In contrast with the previous classifications, here the focus is on the control aspects rather than in the modeling. The approaches that tackle multi-robot manipulation of deformable objects depend usually on singular control laws and complex algorithms. This makes the control-based classification heterogeneous, since a broad variety of approaches is available, and in some cases the approaches cannot be directly assigned to an specific group due to its uniqueness. However, we propose three different control groups in which several studied methods are suitable for being included. Additional groups that can be considered include strategies based on optimal, nonlinear or Jacobian-based control techniques too, but we just include this information in Table I for brevity.

A. Classic control

As long as an approach follows at some level either a proportional or a PID control law, among others, it can be treated as a classical control method. The most recent strategies tend to relegate the classic control techniques to the low-level software layers.

A PD-position feedback controller with gravity compensation is adopted by Sun *et al.* in [28]. This approach considers a general flexible payload whose position and orientation must be controlled by multiple robots, at the same time that vibrations at each contact are suppressed. Deformation of the object is obtained from a finite element model, whose dynamics are decomposed into rigid and flexible components that represent the original undeformed shape and the change in shape due to deformation, respectively. Also Wada *et al.* develop the core of their control strategy in [18] over a classic controller, that in this case is a simple PID control law. They propose two different methods: a model-free PID control system, which is valid only in the domain of small deformations, and a model-dependent PID controller, that provides zero error convergence when the deformation is large. In the latter method a spring mesh approximate model of the object is considered. As opposed to the previous approaches, a classic control method can be found in [24] but at the low-level horizon. Here, velocities of the individual manipulators are controlled by means of a PID control law. This controller may introduce some additional errors in the system when dealing with some types of materials due to a buildup in the integral term. It is important to remark that the use of the classic control law is secondary in this approach, and the main part of the control algorithm (high-level control) is based on advanced planning strategies and convex optimization techniques.

B. Robust control

In robust control techniques, modeling errors and uncertainties are taken into account with the aim of extending the controller's validity. This type of control strategy is well suited when dealing with deformation models due to the fact that uncertainties are always present in the model parameters.

A robust control strategy is considered in [4], where each manipulator's motion is driven by an independent robust controller that is able to work in the presence of model parameters' uncertainties. The global motion planner makes unnecessary any communication between the system agents. Hu and Vukovich develop a shape control system in [29] derived also from the robust control theory. This method aims to produce a desired out-of-plane deformation on a flexible plate with embedded microactuators and sensors, which are represented as a whole in an integrated mathematical model obtained from the Hamilton's principle. In [26] the authors propose a robust passivity-based controller that has into account the presence of a *time-varying* disturbance in the deformation flow estimation, and in [30] the LMI (linear matrix inequality) optimization allows to identify the dynamic parameters of the robotic structure and to define a robust control strategy. In spite of not being a formally developed robust controller, the method proposed by Hu *et al.* in [12] emphasizes demonstrating the robustness of several aspects it covers. One of these aspects is the selection of the state features, in which task-relevant prior knowledge improves the robustness and effectiveness of the control process. Robustness to moderate levels of occlusion, provided that no significant or fast changes happen in the scene, is also achieved thanks to an online learning mechanism.

C. Adaptive control

Uncertainties and errors in the model parameters are also assumed in adaptive control methods, and their values are allowed to change over time in order to adapt to the time-varying systems. Learned deformation model methods are natural candidates for the adaptive control strategies.

An example of the last statement is the adaptive control system by Navarro-Alarcon *et al.* in [11] to estimate the object's deformation parameters. For the arms to move in a coordinated way, a saturated velocity controller is developed here. In the context of working without deformation model, Bai and Wen propose in [19] two different control schemes: a scheme where the robots velocity is predesignated and an adaptive control technique in which the group velocity is estimated by each agent. A special case of the latter technique, in which the group velocity is known by a single agent and the rest have to estimate it, is explained and validated in experimental tests too. However, an adaptive control system can also be adopted when a precomputed deformation model is present. Based on the Potential Field Method, that creates an attractive force to the goal configuration and a repulsive one around obstacles, the approach by Dang *et al.* [31] is focused on controlling the shape of a flexible surface. The surface is modeled as a mass-spring-damper mesh, and a group of embedded microactuators is considered to deform it. These actuators are divided into two different groups: absolutely actuated points, in which information about desired point coordinates is provided, and relatively actuated points, where relative distances to other neighboring points are set. Different dynamic shape morphing adaptive control laws are designed for each group of microactuators including parameters uncertainty.

V. Perception-based Classification

As it can be inferred from the previous section, many different control algorithms are found when analyzing the existing approaches in multi-robot manipulation of deformable objects. In turn, each control method depends on different perception systems. This fact motivates the perception-based classification that we present here, where three main groups are identified in terms of the measured data.

A. Force-based perception

Perception systems focused on forces rely on the fact that interacting with deformable objects necessarily implies a force exchange between all involved agents. For instance, in order to grasp and raise an elastic foam part with a robotic gripper an initial grasping force is required to prevent slipping during the lifting action. Afterwards, a second vertical force must be applied to compensate the part's weight and raise the part. By measuring and controlling these forces, some deformable objects manipulation tasks can be successfully carried out.

This fact is shown in [19], where a force perception system is designed. Their decentralized control method considers the contact forces between the robotic manipulators and the flexible load in order to describe the deformation of the soft object, and also to provide an implicit way of communication to the manipulators. These contact forces are maintained by the controller to avoid sliding of the object during the manipulation task. Delgado *et al.* also consider contact forces by means of tactile sensors in [32], in order to develop an agile and adaptable model-independent multi-robot system for dual-arm in-hand manipulation tasks. They propose a novel representation of the tactile data based on tactile images, which are obtained through a combination of dynamic Gaussians. This representation allows them to design a manipulation controller that maintains and adapts the contact configuration according to the task requirements.

B. Vision-based perception

As the manipulation process evolves, deformation appears and the shape of the object changes. By monitoring these changes with a vision system, the relation between the motion of the manipulators and the deformable object may be obtained, and afterwards the derived model can be utilized by the control algorithm to produce the desired deformation.

Clear examples of vision-based perception systems are included in most of the methods developed by Navarro-Alarcon concerning multi-robot manipulation of deformable objects, like [26], where an energy-based dynamic-state feedback velocity controller is developed. In this approach, deformation of the manipulated object is tracked by a visual feedback system, in which the feedback points are treated with a nonlinear function to constitute a deformation feature vector. Four different types of deformation are defined for the control purposes: point-based, distance-based, angle-based and curvature-based. Calibration of the vision system is not needed here. In [11] the positions of multiple visual markers, which are placed over the surface of the deformable object, are measured with the camera in order to obtain the position and shape errors. This approach can cope with uncalibrated kinematic transformations too. Again, the Fourier-based controller in [10] constantly updates and recalibrates, if necessary, the local deformation model using the vision sensor data. This perceived data also include the full contour of the deformable part, that allows to compute the shape error. One requirement of this method is that a high contrast is needed between the manipulated object and the image background. The recognition system by Li *et al.* in [5] is based on the Kinect sensor. They perform a preliminary 3D segmentation, which obtains the masks of the garments on the depth images, that is followed by the KinectFusion method invocation, which provides the 3D reconstruction. Binary features are extracted from the 3D reconstructed model in order to make the comparison with the models from the database. The Kinect sensor is also utilized by Tang *et al.* in [23] to obtain point clouds of a rope lying on a flat surface. With the aim of manipulating the rope to get some desired configurations, they have designed a multi-robot system based on the CPD (Coherent Point Cloud) non-rigid registration method, which obtains a smooth transformation function from two different point clouds. This method shows strong robustness under occlusions and allows to perform the next three sequential steps: state estimation, task planning and trajectory planning.

C. Force and vision-based perception

By combining the advantages of force and vision perception systems, a more robust and accurate control method may be obtained. In some cases in which a specific tensional state must be induced in the object for the force control to work, the vision system is the one in charge of driving the object to that tensional state. This happens in [25], where a hybrid force-vision controller is proposed. The vision system must drive the manipulated 2D sheet to a taut state, so that the force controller can start to decrease the amount of wrinkles (the system is unable to measure force while the sheet is not taut), by either moving or applying traction to the sheet to counteract the deformations created by the human manipulator. Also in [20] an adaptive force-vision control system is utilized to separate deformable objects. Here, the force controller prevents global deformation or damage in the area around the cut, and the vision system updates online the trajectory according to the sensed deformation and the modeling errors. A different strategy is followed in [24] with a low-level velocity controller that takes into account the forces that are transmitted through the manipulated object. Despite the fact that these forces cannot be sensed (i.e. they are virtual forces), they act in the decentralized planner as an indirect communication channel which is complemented with an inter-agent vision system. This vision system is implemented on each robotic manipulator, and obtains the position and velocity of the neighboring agents. The force controller in [9], which regulates the loading over the deformable object and obtains feedback data from a load cell, is also complemented with a vision system that monitors the object deformation with a camera.

VI. Action-Based Classification

Manipulation of deformable objects involves a sequence of different individual actions that must be performed by the robotic system. These low-level basic actions can be classified into two groups: deformation actions, which consist in inducing relative displacements of the deformable object, and transport actions, which produce absolute displacements of the centre of gravity of the object. Thus, this classification differentiates between methods where the main contribution is provided either by deformation or by transport actions.

A. Deformation actions

Predicting and controlling deformation is usually the foremost concern when manipulating deformable objects due to these reasons: manipulation tasks often require the object to be deformed in an specific way, and also the "unstable" behavior they present may result in damaging the manipulators or the objects themselves in case the deformation is not controlled.

An interesting study about deformation actions can be found in [33]. Cherubini *et al.* propose a vision-based method for deforming 3D materials exhibiting plastic deformation that includes the following initial assumptions: a) it exists a finite set of deformation action types (pushing, tapping and incising) and b) the deformation actions have a local influence into the object. A preliminary study with human participants is performed in order to validate these statements, in which the participants are requested to form a certain shape with kinetic sand in a sandbox, with one or both hands. This process is recorded with a fixed RGB-D camera, and afterwards the output data are collected into a data set which is intended for training neural networks. The study shows that the first assumption is valid, provided that mixed actions also exist, and that the second one has to be relaxed, because actions may affect, in some cases, to the entire state of the material. The assumption of a limited set of manipulation primitives is considered also by Ruggiero *et al.* [30], in the framework of the *RoDyMan* project. This project aims to develop new strategies in robotic nonprehensile dynamic manipulation of deformable objects, with a dual-arm anthropomorphic robotic platform for performing the challenging pizza-making process as the final demonstrator. The pizza-making process includes various deformation subtasks, which they divide into two different nonprehensile manipulation primitives: sliding and tossing. They show diverse results of the project that include a method for real-time tracking of the manipulated object, in which an RGB-D sensor is utilized for obtaining a point cloud that serves to create a finite element model of the object. From a different perspective, Simon and Basri [34] develop a shape matching method for finding a set of deformation actions such that the shape of an initially undeformed surface is transformed into a specific deformed configuration. The initial shape is discretized to a 2D linear elastic finite-element model, that is submitted afterwards to inner condensation in order to reduce the mesh nodes to the ones of the contour, while retaining the physical properties of the rest of the model. A non-linear optimization procedure allows to find the contour forces that produce the desired elastic deformation. It is important to remark that the method is locally defined, which makes it dependent on the source and target shapes initial alignment.

B. Transport actions

Transport actions are necessary when the manipulation task requires the deformable object to be placed on a position which differs from the initial one. High-level tasks usually imply to perform these kinds of actions at some specific instants of the process.

As mentioned before, decentralized control techniques are proposed for transporting a flexible payload by a multi-robot team in [19]. They limit deformation actions to the purpose of providing the required grasping force, while the transport actions have the main role in the tasks. Only the translational problem is considered, leaving the extension to the rotational case open. Also in Alonso-Mora's work the vast majority of the manipulation actions are transport ones. In [24] the considered manipulation tasks relegate deformation actions to avoiding obstacles and overstretching during the ensemble's motion. The same idea is developed by McConachie and Berenson in [14], due to the fact that the purpose is to move the object to a desired position while avoiding obstacles and maintaining the structural integrity of the manipulated piece of fabric. Another interesting transportation case is studied by Sreenath and Kumar in [35], where a box is held in the air through deformable cables attached to a team of quadrotors. The goal of this method is performing feasible trajectories for the quadrotors with the payload, which is simulated by means of a hybrid dynamic model.

VII. Conclusion

From the analysis of the different methods that tackle the problem of manipulation of deformable objects by multiple robots, summarized in Table I, some conclusions may be inferred. Overall, autonomous manipulation of deformable objects is an important and complex problem that is gaining attention in the recent years. It seems clear that using multiple robots is a necessary condition in order to perform the manipulation tasks with flexibility and robustness guarantees, and also to extend the workspace.

Focusing now on the specific solutions we have studied, although model-based approaches can have some advantages in terms of computational cost and accuracy, those approaches based on learning a deformation model in an online manner, or those that directly avoid considering a deformation model, are far more flexible and robust. This flexibility is due to the fact that less assumptions are made with respect to the nature of the object's material and the manipulation system.

Concerning perception methods, hybrid force-vision systems should be chosen to obtain a more complete state of the deformable object. However, depending on the strategy or the system characteristics it may happen that only one type of feedback information is available.

In general, some common issues that manipulation methods are not able to solve yet are the ability to manipulate objects of very different properties with one single strategy and the complete integration between deformation control and transportation of the object. In addition, most of the methods are focused on low-level tasks, and numerous hard constraints are imposed to the systems in order to enable an accurate object's state perception.

Future directions in the field of manipulation of deformable objects by multiple robots include many different topics. Some of these aspects are shared among the aforementioned strategies, and could be summarized as follows: improving deformation sensing systems and deformation models; prediction of future deformation states; automatic computation of the feasible object's configurations and the object's deformation limits; extension of the methods to the 3D space and to different kinds of deformable objects; substitution of fixed robots by mobile robots; and increasing the number of cooperative robotic manipulators.

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TABLE I
METHODS CLASSIFICATION SUMMARY TABLE.

Reference	Model	Object	Control	Perception	Actions	Application	Manipulator
Das *et al.* [4]	Precomputed	2D (rheological)	Robust	-	Deformation	Parts processing	n point robots
Li *et al.* [5]	Precomputed	3D	-	Visual	Deformation/transport	Domestic	Dual-arm robot
Jia *et al.* [6]	Precomputed	2D	Classic	Visual	Deformation/transport	Domestic	Dual-arm robot
Duenser *et al.* [7]	Precomputed	3D	Optimal	-	Deformation	Domestic	Dual-arm robot
Long *et al.* [8]	Precomputed	1D-3D	-	-	-	-	n arms
Higashimori *et al.* [9]	Learned	3D (rheological)	-	Force-vision	Deformation	-	mobile plate and fixed plate
Navarro-Alarcon *et al.* [10]	Learned	2D,3D (elastic)	Adaptive	Vision	Deformation	Medical	cartesian robot and grippers
Navarro-Alarcon *et al.* [11]	Learned	1D-3D (elastic)	Adaptive	Vision	Deformation/transport	Medical	2 robotic arms
Hu *et al.* [12]	Learned	1D,2D	Nonlinear	Vision	Deformation/transport	-	Dual-arm robot
Berenson [13]	Learned	1D,2D	Jacobian	-	Deformation/transport	Domestic	2 floating grippers
Mcconachie *et al.* [14]	Learned	1D,2D	Jacobian	-	Transport	Domestic	2 floating grippers
Langsfeld *et al.* [15]	Learned	2D (elastic)	-	Force-vision	Deformation/transport	Domestic	2 robotic arms
Zhu *et al.* [16]	Learned	1D	-	Vision	Deformation	Parts processing	Dual-arm robot
Alambeigi *et al.* [17]	Learned	3D	Optimal	Vision	Deformation/transport	Medical	n robotic arms
Wada *et al.* [18]	Model-free	2D	Classic	-	Deformation	-	n point robots
Bai *et al.* [19]	Model-free	3D	Adaptive	Force	Transport	Transport	2 or more robotic arms
Long *et al.* [20]	Model-free	3D	Adaptive	Force-vision	Deformation	Parts processing	2 robotic arms
Kudoh *et al.* [21]	Model-free	1D	-	Vision	Deformation	Domestic	Dual-arm robot
Tang *et al.* [22]	Model-free	1D	-	Vision	Deformation	-	2 robotic arms
Tang *et al.* [23]	Model-free	1D	-	Vision	Deformation	-	2 robotic arms
Alonso-Mora *et al.* [24]	Model-free	1D-3D	Classic (low-level)	Force-vision	Transport	Transport	n mobile arms
Kruse *et al.* [25]	Model-free	2D	Jacobian	Force-vision	Deformation	Domestic	Dual-arm robot and human
Navarro-Alarcon *et al.* [26]	Learned	3D (elastic)	Robust	Vision	Deformation	Medical	2 robotic arms
Sun *et al.* [28]	Precomputed	3D	Classic	Pose-velocity	Transport	-	2 robotic arms
Hu *et al.* [29]	Precomputed	2D (elastic)	Robust	Deformation	Deformation	Smart structures	n piezoceramic actuators
Ruggiero *et al.* [30]	Precomputed	3D	Robust	Force-vision	Deformation	Pizza making	Dual-arm mobile robot
Dang *et al.* [31]	Precomputed	2D	Adaptive	Position & velocity	Deformation	-	n point robots
Delgado *et al.* [32]	Model-free	3D	Classic	Force	Deformation/transport	Domestic	Dual-arm robot
Simon *et al.* [34]	Precomputed	2D (elastic)	-	Force-vision	Deformation	-	n point robots
Sreenath *et al.* [35]	Precomputed	1D	Nonlinear	-	Transport	Transport	n quadrotors
Mukadam *et al.* [36]	Model-free	2D (elastic)	Optimal	-	Deformation	-	n floating grippers
Asano *et al.* [37]	Precomputed	3D	-	-	Deformation	Parts processing	n point robots
Tokumoto *et al.* [38]	Precomputed	2D,3D	-	-	Deformation	-	n point robots

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